Problem 1.

Marion says "*the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.*"

In my understanding of the English language, this is false. Let's see an example. Consider a particle in a gravitational field g (pushing towards $-y$), that moves from $y=0$ at time $-t_0$ to $y=0$ at time $+t_0$.

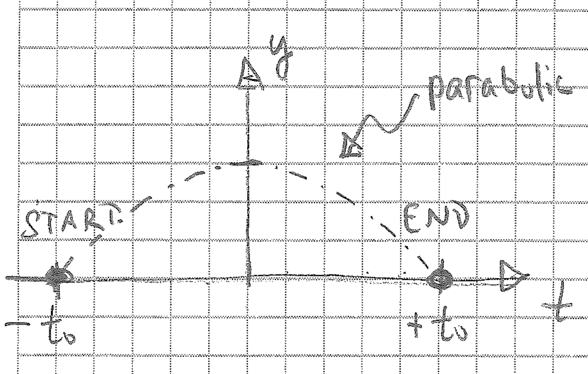
a. What is the path that minimizes the time integral of the kinetic energy?
b. What is the time integral of the potential energy along this path?

I found both of these to be exactly the same. So I am tempted to conclude that this path "*minimizes the time integral of the difference between the kinetic and potential energies.*" But of course, you know this is **not** the correct path... in a gravitational field, objects have parabolic (in time) trajectories.

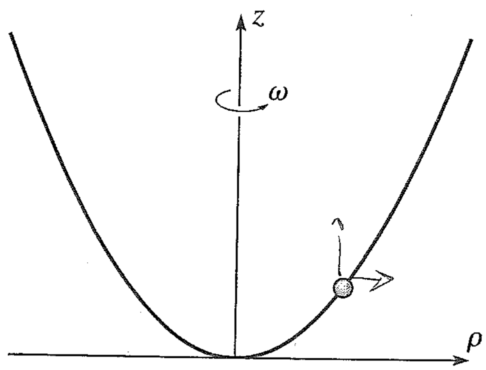
c. Consider a parabolic trajectory given by $y(t)=\alpha(t_0^2-t^2)$ where α is an adjustable parameter. Find the value of α that minimizes $\int_{-t_0}^{t_0} T-U\,dt$ (by direct integration, followed by differentiation wrt α.)

If you feel you must put Hamilton's principle into words, you have two choices:
1)"the actual path followed is that which maximizes the potential energy, while keeping the kinetic energy as small as possible" (this is sort of true, but is a bit vague, since it doesn't tell you to weight both energies the same.)

2)"the actual path followed is that which maximizes the difference between the potential energy and the kinetic energy." This is "Thomas's Principle of Maximum Inaction."



7.41 ★★★ Consider a bead of mass m sliding without friction on a wire that is bent in the shape of a parabola and is being spun with constant angular velocity ω about its vertical axis, as shown in Figure 7.14. Use cylindrical polar coordinates and let the equation of the parabola be $z = k\rho^2$. Write down the Lagrangian in terms of ρ as the generalized coordinate. Find the equation of motion of the bead and determine whether there are positions of equilibrium, that is, values of ρ at which the bead can remain fixed, without sliding up or down the spinning wire. Discuss the stability of any equilibrium positions you find.



7.34 ★★ Consider the well-known problem of a cart of mass m moving along the x axis attached to a spring (force constant k), whose other end is held fixed ([redacted]). If we ignore the mass of the spring (as we almost always do) then we know that the cart executes simple harmonic motion with angular frequency $\omega = \sqrt{k/m}$. Using the Lagrangian approach, you can find the effect of the spring's mass M, as follows: **(a)** Assuming that the spring is uniform and stretches uniformly, show that its kinetic energy is $\frac{1}{6}M\dot{x}^2$. (As usual x is the extension of the spring from its equilibrium length.) Write down the Lagrangian for the system of cart plus spring. (*Note:* The potential energy is still $\frac{1}{2}kx^2$.) **(b)** Write down the Lagrange equation and show that the cart still executes SHM but with angular frequency $\omega = \sqrt{k/(m + M/3)}$; that is, the effect of the spring's mass M is just to add $M/3$ to the mass of the cart.

7-3. A sphere of radius ρ is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius R. Determine the Lagrangian function, the equation of constraint, and Lagrange's equations of motion. Find the frequency of small oscillations.

7-8. Consider a region of space divided by a plane. The potential energy of a particle in region 1 is U_1 and in region 2 it is U_2. If a particle of mass m and with speed v_1 in region 1 passes from region 1 to region 2 such that its path in region 1 makes an angle θ_1 with the normal to the plane of separation and an angle θ_2 with the normal when in region 2, show that

$$\frac{\sin\theta_1}{\sin\theta_2} = \left(1 + \frac{U_1 - U_2}{T_1}\right)^{1/2}$$

where $T_1 = \frac{1}{2}mv_1^2$. What is the optical analog of this problem?

7-16. The point of support of a simple pendulum of mass m and length b is driven horizontally by $x = a \sin \omega t$. Find the pendulum's equation of motion.

7-17. A particle of mass m can slide freely along a wire AB whose perpendicular distance to the origin O is h (see Figure 7-A, page 282). The line OC rotates about the origin

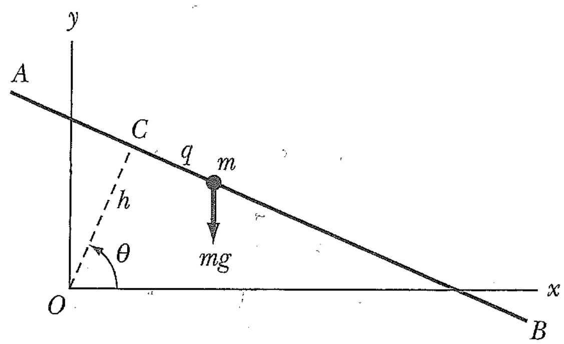


FIGURE 7-A Problem 7-17.

at a constant angular velocity $\dot{\theta} = \omega$. The position of the particle can be described in terms of the angle θ and the distance q to the point C. If the particle is subject to a gravitational force, and if the initial conditions are

$$\theta(0) = 0, \quad q(0) = 0, \quad \dot{q}(0) = 0$$

show that the time dependence of the coordinate q is

$$q(t) = \frac{g}{2\omega^2}(\cosh \omega t - \cos \omega t)$$

Sketch this result. Compute the Hamiltonian for the system, and compare with the total energy. Is the total energy conserved?

7-21. A particle is constrained to move (without friction) on a circular wire rotating with constant angular speed ω about a vertical diameter. Find the equilibrium position of the particle, and calculate the frequency of small oscillations around this position. Find and interpret physically a critical angular velocity $\omega = \omega_c$ that divides the particle's motion into two distinct types. Construct phase diagrams for the two cases $\omega < \omega_c$ and $\omega > \omega_c$.